Department of Mathematics and Computer Science

Friday, November 2, 2018, 4:10 pm

COLLOQUIUM TALK

Speaker: Gregory Galperin (EIU)

Old Main 2210

Projections of Lines in Hyperbolic Geometry

Abstract:

In Euclidean geometry $\mathbb{E}^2$, the orthogonal projection of one straight line, m, onto another straight line, ℓ, covers the line ℓ entirely. In hyperbolic (Lobachevsky) geometry $\mathbb{H}^2$ the situation is drastically different: the orthogonal projection $\mathrm{proj}_\ell(m)$ will never cover the line ℓ entirely! It's either a *finite open interval* or an *open ray*. The first case happens for *intersecting* and *divergently parallel* lines, while the second case, the *ray in the projection*, happens when the lines m and ℓ are asymptotically parallel.

I will prove this statement first without any model of hyperbolic plane, and then, using the Klein model $\mathbb{K}^2$, will derive the formula for the length of the interval in the projection, $|\mathrm{proj}_\ell(m)|$, in terms of either the angle α between the intersecting lines m and ℓ, or in terms of the length p of the common perpendicular in the case when the lines m and ℓ are divergent. During this proof, I will use the Lobachevsky-Bolyai formula for the angle of parallelism φ which I also am going to derive.

In conclusion, I will formulate the intriguing *Bolyai construction* of the angle of parallelism φ, for which I will give my own proof in the Klein model $\mathbb{K}^2$.

All the necessary terms: *angle of parallelism, the Klein model* $\mathbb{K}^2$, *the measurement of distances in the Klein model*, etc., will be introduced and explained during the talk.

SNACKS IN FACULTY LOUNGE AT 3:30 PM.
EVERYONE WELCOME (EVEN IF YOU ARE UNABLE TO ATTEND THE TALK)